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The Helmholtz-Kelvin Time Scale for Very Light Stars

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Abstract

Assuming that the contracting stars in convective equilibrium evolve vertically downwards in the H-R diagram, a simple expression for the Helmholtz-Kelvin time scale $t_{H.K}$ is derived. Application of this expression to stars of mass $M < 0.1M_{\odot}$ shows that these stars contract down a radius of about $0.10R_{\odot}$ in a time scale of less than one billion years while the earlier estimates, based on radiative models, gave a time scale $t_{H.K}$ greater than hundred billion years.

Introduction

Until 1960, it was believed that the time scale for the Helmholtz-Kelvin contraction of very light stars ($M < 0.1M_{\odot}$) is greater than hundred billion years. This belief was based on the assumption that the contracting stars are in radiative equilibrium and this led to a very low luminosity for stars of mass $M < 0.1M_{\odot}$ which, in turn, gave a very long time scale for contraction down to a radius of about $0.10R_{\odot}$. However, recent work by Hayashi (1962) has shown that the contracting stars of low mass remain completely convective during the pre-main-sequence contraction. Therefore, they are much more luminous than completely radiative models and so they evolve much faster. Following this proposal by Hayashi, we shall compute here the time scale $t_{H.K}$ for stars of mass $M < 0.1M_{\odot}$. Before we do that, we shall derive here a simple expression for the time scale for contraction from a radius R_1 to R_2.

Expression for the Time Scale

Hayashi's work and recent work by Ezer and Cameron (1962) show that the completely convective stars contract in such a way that their evolutionary path in the H-R diagram is nearly a vertical line. We shall make use of this vertical evolution to obtain an expression which gives the time scale $t_{H.K}$ quite accurately. The luminosity of a contracting star is given by

$$L = -\frac{\Delta E}{\Delta t} = -\frac{3\gamma-4}{3(\gamma-1)}\frac{d\Omega}{dt} \tag{1}$$

where γ is the ratio of the two specific heats c_p and c_v and Ω is the potential energy of the star. For a completely convective star, which can be represented by a sphere of polytropic index 1.5, Ω is given by

$$\Omega = -q\frac{GM^2}{R} \tag{2}$$

where

$$q = \frac{3}{5-n} = \frac{6}{7} \tag{3}$$

where R is the radius of the star, T_e its effective temperature and σ is the Stefan-Boltzman constant. From equations (1), (2) and (3) we obtain

$$4\pi R^2\sigma T_e^4 = -\frac{3}{7}\frac{GM^2}{R^2}\frac{dR}{dt} \tag{4}$$

In writing down (4), we have put $\gamma = 5/3$. Equation (4) can be rewritten as

$$dt = \frac{3}{28} \frac{GM^2}{4\pi\sigma T_e^4} \frac{1}{R^4} dR \tag{5}$$

We treat T_e as a constant, as the vertical evolution in the H-R diagram means that the star's effective temperature remains constant during contraction. Integrating (5), we obtain

$$t_{H \cdot K} = \frac{GM^2}{28\pi\sigma T_e^4} \left(\frac{1}{R_2^3} - \frac{1}{R_1^3} \right) \tag{6}$$

If we take $R_1 = \infty$, the required expression for $t_{H.K}$ is

$$t_{H \cdot K} = \left(\frac{G}{28\pi\sigma} \right) \frac{M^2}{R_2^3 T_e^4} \tag{7}$$

Expressing M and R_2 in solar units, T_e in thousands of degrees and $t_{H.K}$ in years, we obtain

$$t_{H.K} = 4.98 \times 10^9 \frac{M^2}{T_3^4 R_2^3} \tag{8}$$

For comparison, we give here the corresponding equation for contracting stars in radiative equilibrium (Levee, 1953):

$$t_{H.K} = \frac{2.51 \times 10^7 M^2}{\bar{L}} \frac{1}{R_2} \tag{9}$$

where $\bar{L}$ is the mean luminosity during contraction.

In order to use equation (8), we must know R_2 and T_3 for a star of a given mass. Before we apply it to very light stars, let us apply it to the contracting sun while it remains a fully convective star. Ezer and Cameron (1962) have found that it remains fully convective up to a radius of 3.0 and the time scale up to this stage is 5.0×10^6 years. The mean effective temperature during this contraction is 4300°K and so from equation (8) we have

$$t_{H.K} = 5.4 \times 10^6 \text{ years}$$

This agreement between the two time scales shows that equation (8) is very accurate.

To compute $t_{H.K}$ for very light stars we choose three masses: 0.09, 0.07, 0.05. For each mass we consider the contraction down to the radius R_2 at which the central temperature reaches a maximum value. At this stage a population I star of mass 0.09 becomes a main sequence star while the less massive stars begin to cool towards complete degeneracy (Kumar, 1962). For R_2 we use the radii corresponding to the stage of maximum central temperature computed by Kumar. As far as T_3 is concerned, we use that value which is the lower limit to T_e for a given mass. Table 1 gives the results for the three masses. It is clear that

Table 1

M	R_2	T_3	$t_{H.K}$
0.09	0.11	2.8	4.93×10^8
0.07	0.12	2.5	3.62×10^8
0.05	0.135	2.2	2.16×10^8

the time scales $t_{H.K}$ computed here are very small as compared with earlier estimates of the same quantity. Further, they are very small as compared with the age of the galaxy and therefore we can say safely that there exist a large number of evolved light stars in the galaxy.

Finally, let us apply equation (8) to the star Ross 614B. For this star Limber (1958) gives $R = .0955$, $M = 0.0766$ and $T_e = 2700$. Assuming that $R_2 = R$, which means that either the star is contracting or it stopped contracting at this radius to become a main-sequence star, we obtain $t_{H.K} = 6.31 \times 10^8$ years. In estimating the above time scales, we have neglected the influence of energy generation by the destruction of Deuterium and Li^6 and Li^7. Salpeter (1955) has computed rates for reactions involving these elements and making use of his computations we find that a star of mass 0.09 or .07 will burn all its Deuterium, Li^6 and Li^7 in a time scale smaller than 5×10^8 years. Therefore, these nuclear reactions do not slow down appreciably the Helmholtz-Kelvin contraction. For a star of mass 0.05, the time scale for the destruction of H^2 is small as compared with

2×10^8 years while the destruction of Li^6 may take about 2×10^8 years. This star will, however, not burn any Li^7, as the central temperature never exceeds a value of 2×10^6 degrees. Therefore, the time scale $t_{H.K}$ for this mass may be increased by a factor of tow due to the destruction of H^2 and Li^6.

We conclude that the time scales for the contraction of a very light star to a small radius of about $0.10R_\odot$ is less than one billion years and not hundred billion years or more, as was believed to be two years ago.

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